JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 23, No. 2, March–April 2000

Technical Comments

Brief discussion of previous investigations in the aerospace sciences and technical comments on papers published in the Journal of Propulsion and Power *are presented in this special department. Entries must be restricted to a maximum of 1000 words, or the equivalent of one* Journal *page including formulas and figures. A discussion will be published as quickly as possible after receipt of the manuscript. Neither the AIAA nor its editors are responsible for the opinions expressed by the correspondents. Authors will be invited to reply promptly.*

Comment on "New Design Method for Pulse-Width Modulation Control Systems via Digital Redesign"

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THE design of pulse-width modulation (PWM) control systems has recently gained attention inasmuch as the technology provides increasingly inexpensive devices that can be controlled by on/off commands. The majority of design techniques proposed, the last being the method of Ref. 1, are based on some equivalence principle to an adjoint amplitude-modulated control. In this way PWM control, even if inherently nonlinear, can be studied using linear control techniques if dealing with linear systems. The digital redesign method and PWM input generation proposed in Ref. 1 provides a good agreement of the PWM response with the adjoint continuous time (CT) control response. To quantify the precision of the redesign technique, the authors provide several comparisons, the most appropriate being with the PWM control proposed in Ref. 2 (cited in the referred paper as Ref. 4).

This comparison is not formally correct, because Ref. 2 seeks the equivalence of a PWM control and a discrete time (DT) control, whereas in Ref. 1 the idea is to compare PWM control to a CT control. From the substantial point of view, in this case the CT and the DT control appear so close that PWM performances can be safely compared to both.

It appears that the PWM design of Ref. 2 has been misinterpreted as far as saturation of the pulse width is concerned. In fact, Ref. 1 considers the pulse width saturated when the end of the pulse should be after the sampling instant, and this is explicitly written on page 127 and visualized in Fig. 4b therein.

This interpretation is not correct and was never used in Ref. 2 to establish the equivalence between PWM control and DT control.

As for saturation, Ref. 2 reports on page 463 that "in PWM control the input gets saturated if δ becomes greater than the sampling time Δ, a condition equivalent to the amplitude saturation of the discrete case," where δ indicates the control pulse width. Adopting this saturation concept, the control pulses can "spill" over the subsequent sampling interval, as should be clear from the analysis of Fig. 5 appearing in Ref. 2. This situation is explicitly considered in Ref. 3, where Fig. 4 therein reports the possible conditions of control saturation, partial overlap of pulses, and pulses contained in one single sampling period. This interpretation of PWM saturation is correct and easily translated into control logic, as documented by the experimental results presented in Ref. 4. These results, reported in Figs. 9 and 10 on pages 239–240 (Ref. 4), clearly show that the pulse duration can be greater or equal to the sampling time for several consecutive sampling intervals, with no need to switch the control off at the sampling instants.

Received 16 April 1999; accepted for publication 8 September 1999.

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A second dubious interpretation of Ref. 1 concerns the steady-state response of the PWM control of Ref. 2. Figure 4a of Ref. 1 shows a steady-state response different from the command, even if the control input is far from the supposed saturation of half the sampling period. In Ref. 2, instead it was proven [Eqs. (18–27)] that the PWM steady-state response is on average equal to that of the DT control, regardless of the pulse delay, provided the control does not saturate. This should be the case of the example of Ref. 1.

A further interesting feature of the PWM of Ref. 2 concerns the possibility of taking into account the computational delay. If the computational delay is known, the fixed control delay can be reduced correspondingly to provide a more precise control. This was not considered in Ref. 1.

The interested reader may find other methods to translate a DT control into a PWM control. Reference 5 establishes the equivalence using the principle of equivalent area to generate a single pulse centered within the sampling period. Reference 6 refines the method to provide a higher-order equivalence of the time response by dividing the pulse area into several shorter pulses spread over the sampling interval; this method is called multipulse-width modulated control (MPWM).

By the correct use of the method of Ref. 2, the response of Fig. 4 in Ref. 1 should appear as in Fig. 1 herein. For comparison, the CT control, the digitally redesigned DT control of Ref. 1, the PWM response of Ref. 2, and the PWM response of Ref. 5 have been reported. Notice that all controllers provide a satisfactory time response, with differences between CT, DT, and PWM control that are barely visible.

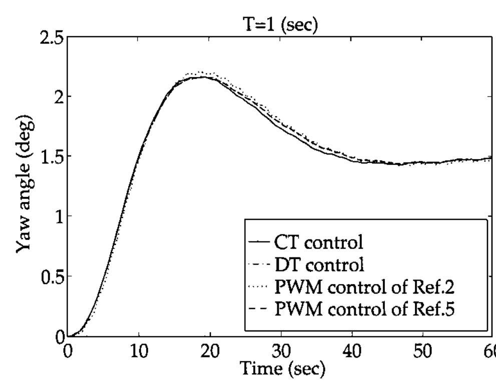

Fig. 1a Time histories of yaw angle, $T = 1.0$ s.

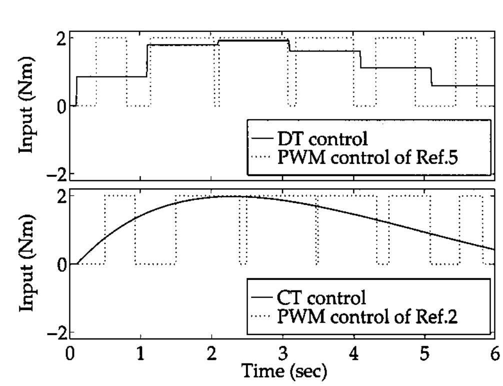

Fig. 1b Time histories of PWM input, $T = 1.0$ s.

The method of Ref. 2 provides a much better agreement with the CT control than supposed by Ref. 1. The method of Ref. 5 appears precise enough and comparable to the one proposed in Ref. 1. For this reason, and for conciseness, the MPWM strategy of Ref. 6 has not been considered.

References

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[5]Shieh, L., Wang, W., and Sunkel, J. W., "Design of PAM and PWM Controllers for Sampled-Data Interval Systems," *Journal of Dynamic Systems, Measurement, and Control*, Vol. 118, No. 4, 1996, pp. 673–682.

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Reply by the Authors to Franco Bernelli-Zazzera and Paolo Mantegazza

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As the comment states, cutting off a pulse at the sampling instant $t + \Delta$ when the pulse width is greater than $\Delta - \tau$ is not consistent with the principle of equivalent area (PEA). We overlooked this point and did not find the overlap method employed in the simulation in Ref. 1; however, it is not explicitly mentioned in Ref. 1. Although the overlapping is consistent with the PEA, it is not a theoretical result. In fact, the optimal delay time is derived from the state-error analysis at the sampling instant $t + \Delta$, where the pulse width is assumed to be smaller than $\Delta - \tau$, that is, no overlapping is assumed. Therefore, there is no guarantee that the method always works well, thought it may give better results than cutting off the pulse. As pointed out in Ref. 2, the method of Ref. 1 "does not work well for the systems that operate with smaller control signals, which require longer pulse durations." In such a case, the firing time often overlaps with the next sampling interval. The reason for the poor performance is that in Ref. 1 $\Psi(-\delta)$ is approximated by $\boldsymbol{I}$, whereas the method of Ref. 2 and ours equivalently approximate it by $\boldsymbol{I} - \boldsymbol{A}\delta/2$; thereby, the better delay time is obtained as well as the pulse width that is strictly consistent with the PEA. The authors of Ref. 1 later improved their method in Ref. 3. As a matter of fact, the delay times provided by their new method in the one-pulse case and by the methods of Refs. 2 and 4 are exactly the same. Reference 2 first presented the delay time. Unfortunately we did not know the paper. However, now that the better delay time is found, the delay time of Ref. 1, hence the overlap method, will not be used. However, we admit that the overlap method works much better than the pulse cutoff.

We reply to the other points of the comment as follows.

1) As to the steady-state error in Fig. 4a of Ref. 4, because the computed pulse width is actually greater than $\Delta - \tau$ and we cut off the pulse at the next sampling instant $t + \Delta$ or more precisely $t + \Delta$ plus computational delay, the PEA does not hold for the pulse. This means that the control power is not enough to reproduce the state responses of the [pulse-amplitude modulation] PAM control system. For this reason, the steady-state error remains. The authors of Ref. 1 state in the comment that "the [pulse-width modulation] PWM steady-state response is on average equal to that of [discrete time] DT control, regardless of the pulse delay. . . . " However, this statement may lead to a misunderstanding because on average the PWM control does not always achieve no steady-state error. In fact, in the case of no input delay in Fig. 3 (Ref. 4), where neither firing time saturation nor overlap occurs, the steady state eror exists. The steady-state analysis shown by Eqs. (18–27) indicates that if the state responses of the PAM control system agree with those of the PWM control system at every sampling time in the steady-state, then the preceding statement holds. However, the analysis does not show that the PWM control brings the system to the same steady states as those that the PAM brings it to. The steady-state output error may disappear, if the PAM controller or the plant has integral action. This is the case with the example of Ref. 4; however, zero steady-state error is not achieved due to the pulse cutoff in Fig. 4a.

2) As to computational delay, because the method of Ref. 1 gives a constant delay time, the effect of computational delay can be alleviated by reducing the firing delay time by the computational delay time. Also in the method of Ref. 4, however, when the delay time given by Eq. (21), which is variable, is larger than the computational delay, the same logic is applicable. Even when it is smaller than the computational delay, the effect of the delay can be minimized by firing a pulse immediately after the computational delay.

In any case, we admit that cutting off a pulse results in the worse evaluation than should be by taking the overlap method. We also regret not having referred the reader to Refs. 2 and 3 that derived the same delay time as Ref. 1. We appreciate their comment.

References

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Received 16 April 1999; accepted for publication 8 September 1999.

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